A Practical Iterative Price-Based Approach for Optimal Demand-Response

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Abstract— **Demand Response (DR) programs encourage consumers to adjust their power consumption in response to DR events such as changes in electricity prices or sudden peaks in demand. While significant savings can be thus achieved, the real success of DR programs depends on the incentive compatible participation of consumers and their timely response to DR signals. In this work, we consider households that operate different types of appliances consuming electricity loads characterized by preferences on the time and quantity of the consumption. We develop a distributed optimization algorithm of practical interest, which offers both to the providers and the consumers of electrical power the opportunity to jointly interact and converge to the optimal scheduling of their appliances: this optimizes the trade-off between user-savings with the inconvenience caused by shifting consumption in time and shading load, while leading the cost of the utility company to the socially optimal level. Our approach is based on related work, and solves two key practical issues: i) we convert the consumer reaction problem to the solution of an LP using standard LP solvers, and ii) we propose a heuristic for load consolidation, i.e., regroup appliance operation on adjacent timeslots, using a randomized algorithm. We also present a substantial amount of simulation data that suggests that our heuristics are sound and reach quickly a state relatively close to the actual system optimum.**

Keywords-Demand response; dynamic pricing; energy consumption scheduling; incentives; smart grid

I. INTRODUCTION

Demand Response (DR) programs encourage electricity end-consumers to adjust their consumption in response to DR events, such as changes in electricity prices surges in demand. Energy use in buildings, including commercial and public buildings, represents approximately 40% of total final energy consumption and 36% of CO2 emissions in Europe [1]. Space heating accounts for 67 % of household energy consumption in the EU27 and is followed by water heating (14%) and appliances/lighting (13%) [2]. Therefore, one expects significant savings to be achieved by DR. Although DR programs have been implemented in industrial environments so far, their penetration in the residential sector can result in considerable savings, due to the fact that such environments account for a large portion of the total energy demand. However, the real success of residential DR programs depends on the incentive compatible participation of consumers and their timely response to DR events.

In this work, we focus on the derivation of electricity prices for different times in a given time frame (day or week or month) by means of a mechanism that solves the social (global) welfare optimization problem, i.e. maximizes the aggregate benefit to the consumers minus the cost of provisioning. The idea is very simple and is borrowed from [3].

It defines a market for electricity consumed at different times, while prices are set to marginal costs. Since such goods are substitutable, the right prices offer the incentives for shifting and shading consumer demand, and at the equilibrium we obtain economic efficiency. Our contribution is to design the consumer response algorithms in a way that leads to simple implementations and that also capture practical constraints on the appliance side, which are hard to capture in the microeconomic model of the market proposed by [3].

Our approach presumes an infrastructure supporting consumption data gathering and processing. This information is used to reveal each consumer's preferred consumption pattern and her disutility for changing it. It has to take into account historical demand behaviours and other data revealing the context that affected the consumption (weather conditions, social events, etc.). Utilizing this knowledge, the provider may estimate the right DR signals, e.g., prices at a given time zone or timeslot. Our mechanism can be applied under multiple contexts in a straightforward manner; e.g. we differentiate weekends from weekdays, and run two separate interleaved instances of our algorithm.

Many approaches in the literature, see [2], [5-9], define algorithmic approaches that mostly focus on solving each consumer's personal optimization problem. For instance, [2] proposes a residential energy consumption scheduling framework, which attempts to achieve a desired trade-off between minimizing the electricity payment and minimizing the waiting time for the operation of each appliance in household under a real-time pricing tariff, using price prediction based on prior information. Also in [9] the authors propose a multifunctional system named "Yupik", which solves the user-side optimization problem and presents the right choices to the users by using an appropriate interface. The model involves simple linear disutility functions for delaying the scheduling of certain appliances, with parameters obtained by monitoring the history of the operation of these appliances.

Solving the global optimization problem involves the participation of the utility company. By introducing a distributed global optimization algorithm, [3] offers both the utility company and the consumers the opportunity to jointly compute the optimal prices and the demand schedules for these prices, by means of an iterative procedure. The model considers households that operate different appliances including PHEVs and batteries; the appliances are divided into types according to their operational characteristics and are described by concave utility functions and linear constraints.

Our contribution is in remedying some of the shortcomings of the approach in [3]. One concerns the complexity in solving the optimal load schedule selection from the part of the consumer. Although the problem is the one of maximizing a set of concave functions over a set of linear constraints, this requires in general the use of computationally complex gradient projection methods. We propose a simpler way to solve this problem exploiting the special property that the optimum is always on the boundary of the constraint set. This involves the local linearization of the utility functions and the use of a standard LP solver, which allows us to easily handle a general form of linear constraints. LP solvers are remarkably efficient in practice and their worst case exponential complexity is acceptable for small scale residential problems.

A second problem concerns the micro-economic modelling of appliances like washing machines, water heaters, etc, which can shift load between different timeslots. Some of them like the water heaters can consume electricity load on non-adjacent timeslots. However, other appliances like washing machines, need their consumption to be better consolidated and use a single or more adjacent timeslots. The mathematical analysis of a market where there are such combinatorial constraints on the consumption of goods is not a standard one and we don't know of simple iterative methods that work. This is the reason why our basic appliance models cannot address such load consolidation constraints and hence stay in the "computationally effective" world of simple linear constraints. This has always been a problem in models such as in [3], which trade modelling accuracy with tractability.

To tackle the above problem, we deploy a simple heuristic. Say that we have *n* consumers of the same type, which need to schedule an appliance (say a dishwasher) that requires consolidation (must run in a single time slot), and the optimal schedule for each consumer is to use 2 units of load in timeslot 1 and 1 unit in timeslot 3. Suppose we let each consumer randomly choose among the slots 1 and 3 with probabilities proportional to the required loads, i.e., 2/3 and 1/3, and run the appliance in this chosen slot using the total of the scheduled load (in this case 3 units). Then for large *n*, by the law of large numbers, at each time slot the expected value of aggregate load generated by the *n* consumers would be the same as the aggregate load in the non-randomizing case, with a relatively small variance. This load consolidation heuristic is transparent to the utility company and also offers practical schedules for the consumers. We have resorted in extensive simulations experiments to assess the speed of convergence and the efficiency of the resulting equilibrium. Our results suggest that our method has excellent convergence properties and also leads to essentially the same social optimum as the exact model, with negligible variations.

This paper is organized as follows. In Section II we present the basic model for the utility company and the consumers and in Section III the models for appliances and the associated utility functions, all based on [3]. In Section IV we present our heuristics for consumers' reaction and in Section V the experimental results validating our approach. Finally, in Section VI, we give some concluding remarks.

II. THE MODEL

As we mentioned earlier, our approach is based on the work in [3]. We present below the complete model borrowing the notation from the above paper. We describe how the utility company sets its prices dynamically, how each consumer responds, and the properties of the resulting equilibrium operating point. The basic idea is to split time into time slots and create a market for electricity in each slot. At the market equilibrium we achieve economic efficiency as expected. We exploit the inherent distributed decision-making that such a market model offers.

To make things more concrete, one can think that our basic frame of time consists of a single day. As we just mentioned, each day is divided into T timeslots of equal duration (say hours), indexed by $t \in T \coloneqq \{1,2,\dots,T\}$. During each day the seller (utility company) posts prices for the different slots and the consumers choose their loads. This process iterates daily until equilibrium is reached. We use k to denote the kth day of the iterative process.

A. System Model

Consider a set N of households/consumers that are served by a single utility company. Our system consists of the utility company and the consumers.

1) The Utility Company:

The utility company serves as an intermediary that participates in multiple wholesale markets, including day-ahead, real-time balancing and ancillary services, to provision enough electricity to meet the demands of the N consumers. We assume that cost of electricity to the utility company is summarized by the cost function $C(Q,t)$ that specifies the cost to provide Q amount of power to the N consumers at time t, where the cost function $C(Q,t)$ is taken as convex and increasing in Q for each t. The utility company sets the prices $(p(t), t \in T)$ as the marginal cost at the total consumption level; the optimal such level and the corresponding prices are discovered by means of an algorithm described below. Setting prices equal to marginal costs does not guarantee complete cost recovery by the utility company. We implicitly assume that such issues are solved by a fixed part in the user tariffs.

2) The Consumers

Each consumer $i \in N$ operates a set A_i of appliances such as air conditioner, refrigerator, plug-in hybrid electric vehicle (PHEV), etc. (In [3] it is taken that consumers can also operate batteries and store power; for simplicity, we do not take this feature into account.) For each appliance $a \in A_i$ of consumer i, $q_{i,a}(t)$ denotes its load drawn at time $t \in T$ and $q_{i,a}$ the vector $(q_{i,a}(t), t \in T)$ of loads drawn over the whole day. An appliance a is characterized by two parameters:

- A utility function $U_{i,a}(q_{i,a})$ that quantifies the utility consumer i obtains when it consumes power according to the vector $q_{i,a}$; and,
- a set of linear inequalities $A^{i,a} q_{i,a} \leq \eta_{i,a}$ on the power vector $q_{i,a}$ corresponding to technical constraints.

In Section III we will describe in detail how [3] models various appliances through appropriate matrices $A^{i,a}$ and vector $\eta_{i,a}$. Note that an inelastic load, e.g. minimum refrigerator power, can be modelled by $q_{i,a}(t) \geq \underline{q_{i,a}}$, which implies that the appliance a of consumer i requires a minimum power $\underline{q_{i,a}}$, at all times t. This is a linear constraint and part of the set of constraints $A^{i,a} q_{i,a} \leq \eta_{i,a}$.

At each time t the total power demand of consumer i is

$$Q_i(t) \coloneqq \sum_{a \in A_i} q_{i,a}(t). \tag{1}$$

3) System Problem

Suppose a central planner has the objective to maximize the economic efficiency of the complete system consisting of the utility company and the consumers. Then she would solve

$$\max_q \quad \sum_i\left(\sum_{a \in A_i} U_{i,a}(q_{i,a})\right) - \sum_t C\left(\sum_i Q_i(t)\right) \tag{2}$$

$$\text{s.t.} \quad A^{i,a} q_{i,a} \leq \eta_{i,a}, \ \forall a,i \tag{3}$$

$$0 \leq Q_i(t) \leq Q_i^{max}, \ \forall t,i \tag{4}$$

where $Q_i(t)$ is defined in (1), the inequality (3) models the various consumer appliances (see Section III for details), the right-hand inequality of (4) imposes a bound on the total power drawn by consumer i at each slot.

By assumption the objective function is concave and the feasible set is convex, and hence there is a unique optimal point, which can in principle be computed centrally by the utility company. This, however, would require the utility company to know all the consumer utility and cost functions and all the constraints, which is clearly unrealistic.

A simple property of the optimal solution of (2) is that if each consumer was charged at each slot the marginal cost of the cost function evaluated at the optimal load vector, then she would also consume the amount of optimal load specified by (2). This suggests a distributed solution of (2) in terms of a market mechanism where the utility company posts prices and the consumers react by changing their demands in each slot in order to maximize their net benefits. At the *equilibrium* we expect the utility company to post prices $p := (p(t), t \in T)$, which induce the consumers to individually choose the consumptions q_i that correspond to the solution of (2), and these prices are the marginal costs that correspond to the aggregate of the above consumptions.

4) Distributed Algorithm

The distributed algorithm involves iterations between the consumers and the utility company.

a) Utility company: in step of the iteration, the utility company posts the price vector p that corresponds to the marginal cost of the aggregate load vector of the previous step.

b) Consumers: During each iteration, given the prices p posted by the utility company, each consumer i chooses her own power demand for all her appliances $q_i := (q_{i,a}(t), \forall t, \forall a \in A_i)$ so as to maximize her net benefit; namely, the difference of the total utility of operating each appliance a at power levels given by the vectors $q_{i,a}$ minus the total cost of electricity.

To avoid oscillations, *each consumer moves slowly her consumption in the direction of the optimal consumption*, i.e., she does not consume this optimal amount immediately but chooses an amount that corresponds to a weighted average of her old consumption and the optimal one. If this does not belong to the feasible set defined by the inequalities (3) and (4), then a projection of the aforementioned weighted average to this set is ultimately taken. It is also realistic to assume that consumers would not switch directly to the optimal consumption schedule but rather take it into account in modifying their previous schedule. An alternative approach to avoid oscillations, is to have consumers use their optimal load and prices posted by the utility company move slowly in the directions of the true marginal costs. This latter case might not be acceptable by the utility company because it may not recover its costs in the case of simple linear cost structures.

Therefore, during each iteration consumer i solves her *local problem*:

$$\max_{q_i} \sum_{a \in A_i} U_{i,a}(q_{i,a}) - \sum_t p(t) Q_i(t) \qquad (5)$$

$$\text{s. t.} \quad (3) \text{ and } (4) \text{ apply.}$$

Note that the optimal solution of each consumer i depends on the prices $p := (p(t), t \in T)$ set by the utility company. Thus, it is denoted by $(q_i(p)) := (q_{i,a}(t; p), \forall t, \forall a \in A_i)$.

The prices p and the total consumer demand $q := (q_i, \forall i)$ are in equilibrium if $q = q(p)$, and $p = C'(q)$. At this point, we know from the first order conditions of (2) that the social welfare given by (2) is also maximized. One can show that starting from an arbitrary initial operating point, the iterative procedure mentioned earlier alternating between the utility company and the consumers converges to the unique solution of (2). As already mentioned, an important requirement for convergence is that consumers don't do abrupt load changes and move slowly towards their optimal choices.

III. APPLIANCE MODELS AND INTEGRITY CONSTRAINTS

We briefly discuss the appliance models proposed in [3]. The basic idea is to model appliances as accurately as possible while preserving the computational tractability of the resulting optimization problem, namely the concavity of the function to maximize and the linearity of the constraints.

The consumer appliances are classified into four types; each type is characterized by a utility function $U_{i,a}(q_{i,a})$ that models how much consumer i values the consumption vector $q_{i,a}$, and a set of linear constraints on the consumption vector $q_{i,a}$, which are all involved in the optimization problems defined in Section III. Below, we mostly focus on the so-called type 2 of appliances, which exhibit important load-shifting aspects and their accurate modelling is hard (see our following discussion on this topic).

In particular, this type includes the appliances such as PHEV, dishwasher and washing machine. For these appliances, a consumer only cares about whether the task is completed before a certain time. This means that the cumulative power consumption by such an appliance must exceed a threshold by the deadline (e.g. see [4]). Let $A_{i,2}$ denote the set of type 2 appliances of consumer i. For each $a \in A_{i,2}$, $T_{i,a}$ is the set of times where the appliance can operate. For instance, for PHEV, $T_{i,a}$ is the set of times that the vehicle can be charged. For each consumer i and each appliance $a \in A_{i,2}$, we have the following constraints on the load vector $q_{i,a}$:

$$q_{i,a}^{min}(t) \le q_{i,a}(t) \le q_{i,a}^{max}(t), \qquad \forall t \in T_{i,a} ,$$
$$q_{i,a}(t) = 0, \ \forall t \in T \setminus T_{i,a} ,$$
$$\bar{Q}_{i,a}^{min} \le \sum_{t \in T_{i,a}} q_{i,a}(t) \le \bar{Q}_{i,a}^{max} , \qquad (6)$$

where $q_{i,a}^{min}(t)$ and $q_{i,a}^{max}(t)$ are the minimum and maximum power loads that the appliance can consume at time t, and $\bar{Q}_{i,a}^{min}$ and $\bar{Q}_{i,a}^{max}$ are the minimum and maximum total power draw that the appliance requires. By setting $q_{i,a}^{min}(t) = q_{i,a}^{max}(t) = 0$ for $t \in T \setminus T_{i,a}$, we can rewrite the constraints of (6) as

$$q_{i,a}^{min}(t) \le q_{i,a}(t) \le q_{i,a}^{max}(t), \qquad \forall t \in T$$
$$\bar{Q}_{i,a}^{min} \le \sum_{t \in T_{i,a}} q_{i,a}(t) \le \bar{Q}_{i,a}^{max} \qquad (7)$$

The overall utility that the consumer i obtains from a type-2 appliance a depends on the <u>total power consumption</u> by a over the whole day. Hence the utility function in the form introduced in Section II reduces to:

$$U_{i,a}(q_{i,a}) := U_{i,a}(\textstyle\sum_t q_{i,a}(t)) := U_{i,a}(Q_{i,a}) , \qquad (8)$$

where $$Q_{i,a} = \sum_t q_{i,a}(t) . \qquad (9)$$

It is assumed that this utility function is a continuously differentiable concave function of $Q_{i,a}$.

A key observation is that the above model satisfies the key properties required for solving it effectively (concavity of the utility function, linear constraints), but it is not realistic for most typical devices we like to model since it allows for such a device to be scheduled in non-adjacent slots and use a fraction of the total load in each slot. For instance, a washing machine requiring 2 units of load could be scheduled to use 2/3 units of load in slots 1, 3, 5. This might be practically impossible. Hence in practice, type 2 devices may have *integrity constraints*. These constraints dictate that power consumption should be in adjacent slots, i.e. the set of timeslots $T_{i,a}$ where the power is positive should constitute a single time interval. This is a combinatorial requirement and clearly destroys the linearity of our constraints making the computational solution very expensive.

Our methodology to solve this problem is a heuristic that uses the solution of the problem without these integrity constraints, and

at each step it aggregates the load at each consumer's premise in a way that looking at the aggregate load generated by all consumers it remains essentially the same without the consolidation. For simplicity we assume that each type 2 device needs to be scheduled in a *single* slot. This is a realistic assumption for residential devices, as each slot's duration offers adequate time for the accomplishment of their task. As we discuss in detail in the relevant section, each consumer picks a slot at random with probability proportional to the components of the load vector in the solution of (5) and runs the device at that slot using the total load.

Our experiments also comprise the so-called type 3 of appliances, which includes appliances such as lighting that must be on for a certain period of time. A consumer cares about how much light can get at each time t. Thus, user utility from a particular consumption vector $q_{i,a}$ is the sum of utilities due to the consumption in each timeslot; that is $U_{i,a}(q_{i,a}) := \sum_t U_{i,a}(q_{i,a}(t),t)$, subject to constraints on minimum and maximum power loads that the appliance can consume at time t.

IV. HEURISTICS FOR CONSUMER REACTION

The local problem to be solved by each consumer is the maximization of a concave function with linear constraints, and is traditionally solved by a gradient projection type of algorithm. This makes sure that in all iterations we move without violating constraints and stay in the interior of the feasible region. In the case of general linear constraints these algorithms are computationally expensive to run. In [3] the gradient projection algorithm used in the numerical experiments is simple (we can consider each appliance individually) because the constraints across appliances (given by (4)) do not appear to be taken into account, and thus the problem obtains a special decoupling structure. This is not anymore the case if we do consider constraints on the total power consumed by all appliances at each slot.

Our idea for addressing such general constraints is as follows: at each iteration, we use a standard LP solver to find the optimum of the linearized problem of the consumer at the previous operating point. Then, we derive the updated consumption schedule by moving in this direction using a decreasing step size. This simple computational procedure works well and converges to the optimum, which in our case is always on the boundary of the simplex.

Instead of adopting the above sequence of solutions, we introduce an approximation by means of our load consolidation heuristic mentioned previously. In particular, in order to satisfy the practical integrity constraints and group the load consumption of type 2 appliances in a single slot, we use a randomization procedure that alters the load profiles proposed by the optimization algorithm. The law of large numbers suggests that in the case of many consumers of similar types, the aggregate load after the randomization has small fluctuations around its mean, which is the non-randomized aggregate load. Hence, when enhanced with this randomizing heuristic, our algorithm displays similar convergence properties and converges to the same equilibrium. Note that the heuristic can also be combined with the approach of [3], which does not conform to the practically applicable load integrity constraints.

Note also that these two heuristics may be applied <u>either unilaterally or together</u>, as each one of them offers a discrete benefit to the optimization procedure. However, the load consolidation heuristic may not be always feasible to be applied on each occasion, as it may violate the constraints across the appliances (see Section V-C). We summarize now these ideas in detail. In what follows, k denotes the iteration step.

A. Linearization of Concave Utility Functions

To simplify the presentation, we restrict attention to appliances of type 2. The utility function that represents the benefit obtained by

their use is given by equations (8) and (9). The local problem is again the optimization of each consumer's net benefit. However, at each iteration k, we resort to a linear approximation of this concave utility function that depends on the current operating point, thus obtaining:

$$\tilde{U}_{i,a}^k(q) = \gamma_{i,a}^k q \ , \tag{10}$$

where $\gamma_{i,a}^k$ is the slope of the concave utility function $U_{i,a}(.)$ at the total consumption of this appliance at current iteration.

Therefore, the consumer's problem that we solve at each iteration k is as follows:

$$\max_{q_i} \quad \sum_{a \in A_i} \tilde{U}_{i,a}^k(q_{i,a}) - \sum_t p^k(t) Q_i^k(t) \tag{11}$$

s. t. (3) and (4) apply.

Let $\tilde{q}_{i,a}^{*k}$ be the solution of the LP (11). We use "tilde" to denote the quantities related to the linearized local solution. The new consumption vector will be now computed as:

$$\tilde{q}_{i,a}^k = g^k * \tilde{q}_{i,a}^{*k} + (1 - g^k) * \tilde{q}_{i,a}^{k-1} \ , \tag{12}$$

where $g^k = \frac{g}{\sqrt{k}}, \ g < 1.$

The selection of g influences how fast we move in the direction of the optimal load selection from the linearized local problem. As we discussed before, the actual load consumption proposed by our method is obtained by applying the consolidation algorithm to the solution $\tilde{q}_{i,a}^k$, as explained next. The formulation of the local problem as an LP (11) allows us to use a standard LP solver and thus to easily handle the various linear constraints. Therefore, we maintain feasibility in each iteration, contrary to the outcome of the gradient projection method of [3], which if necessary is projected within the feasible set at the end of each iteration.

B. Load Consolidation

Each consumer i chooses independently from others to sum-up the aggregate consumption $\tilde{Q}_{i,a}^k = \sum_t \tilde{q}_{i,a}^k$ for each type 2 appliance he operates, at one and only slot t. This slot is selected randomly; in particular, the probability that slot t is selected is $P_{i,a}^k(t)$, which equals the ratio of consumption $\tilde{q}_{i,a}^k(t)$ at this slot over the aggregate consumption $\tilde{Q}_{i,a}^k$, i.e. $P_{i,a}^k(t) = \frac{\tilde{q}_{i,a}^k(t)}{\tilde{Q}_{i,a}^k}, t = 1,2,\dots,T$. Hence, the actual consumption of each type 2 appliance a of a consumer i is zero in all slots except for the selected slot where the consumption equals $\tilde{Q}_{i,a}^k$. Let $\hat{q}_{i,a}$ denote the resulting loads. We use "hat" to denote the load quantities after linearization *and* load consolidation. The actual proposed reaction of consumer i corresponds to $\hat{Q}_i^k$ and we assume that a consumer always follows the above proposed schedule. Then, the utility company computes and posts its prices for the next iteration as the marginal cost corresponding to this consumption, i.e. $p^{k+1}(t) = C'\left(\sum_i \hat{Q}_i^k(t)\right)$.

The heuristic should be applied only if load consolidation does not lead to violation of the bound on the maximum total consumption in any timeslot t, which we assume in the sequel.

V. NUMERICAL EXPERIMENTS

In this section, we present the results of a variety of experiments to illustrate the application of the proposed methodologies. First, we present the basic input parameters for the experiments, which in certain cases are partly differentiated, as explained accordingly.

Consider a system with N households (consumers) entering a DR program. Each household is assumed to have one type 2 and

one type 3 appliances, e.g. washing machine and lighting. Each type 2 appliance may operate at each slot and as already mentioned the respective consumer is interested in the aggregate consumption throughout the whole day. For the type 3 appliances, consumers are only interested during night time. As in the experiments of [3], each day is divided into 8 timeslots of equal duration (i.e. 3 hours), with the first slot corresponding to the time-zone 8-11am. Thus, type 3 appliances only operate in the slots 5-8. The basic parameters used in simulations are as follows:

For each type 2 appliance, $\bar{Q}_{i,A2}^{min}$ and $\bar{Q}_{i,A2}^{max}$ are chosen randomly and uniformly from [1400Wh, 1600Wh] and [2400Wh 2500Wh] respectively, while the values of $q_{i,A2}^{min}(t)$ and $q_{i,A2}^{max}(t)$ are 0Wh and $\bar{Q}_{i,A2}^{max}$ Wh respectively. The utility function is assumed to be logarithmic of the form:
$U_{i,A2} = w_{i,A2} \ln(Q_{i,A2})$, where $w_{i,A2}$ is randomly and uniformly distributed in the range [7700 , 12320] for each consumer i.

For type 3 appliances, the minimum and maximum per slot working power requirements are 200Wh and 800Wh respectively. The utility function is assumed to be logarithmic of the form:
$U_{i,A3}(t) = w_{i,A3}(t) \ln(q_{i,A3}(t))$, where $w_{i,A3}(t)$ for each consumer i, is randomly and uniformly distributed in the range [3800, 3900] for $t = 5,6,7,8$, while $w_{i,A3}(t) = 0$, $t = 1,2,3,4$.

We also assume that the electricity cost function is of the form:
$C(Q) = cQ^2 + bQ + a$, thus $p(t) = C'(Q(t)) = 2cQ(t) + b$ and we use the values $c = 1/200N$ and $b = 0.8$ for each experiment. Except for Section V-C, we do not consider an upper bound on the total consumption per household (given by (4)).

A. Evaluation of the Linearization Method

First, we apply the linearization method (only) and evaluate it compared to the algorithm introduced in [3], with regard to: the convergence state, the iterations needed to approach it and the social welfare attained at each iteration towards the optimal. For each method, we choose different proper values for parameter g. For the algorithm in [3] we pick $g = 30$ such that it convergences within a reasonable number of iterations (order of 10). For our approach we choose $g = 0.25$ for iterations $k = 1,2,…,100$ in order to move fast towards the optimum and a lower value, $g = 0.03$, for iterations $k = 101,102,…,200$ in order to "smooth-out" the oscillations around the optimal state.

We assume a system with $N = 10$ households participating in the DR program. The initial aggregate consumption of each type 2 appliance is assumed to be allocated throughout the whole day, but biased proportionally to the number of the slot, meaning that it causes peak-demands. For instance, $\tilde{Q}_{A2}^0(8) \approx \frac{8}{7} * \tilde{Q}_{A2}^0(7)$. The initial consumption for type 3 appliances is randomly and uniformly distributed in the range [580Wh, 600Wh] for slots $t = 5,6,7,8$, while it has zero value for slots $t = 1,2,3,4$.

Fig. 1 indicates that at the convergence state, the two algorithms exhibit similar behavior with regard to the aggregate (system) per slot consumption. They distinguish the energy consumption into two parts during the day. The type 2 appliances, which can be "ON" over the entire day, gradually shift their load to the first four slots, during which type 3 appliances gain no benefit from consumption. Despite their convergence at the same optimal state, their behavior differs during iterations in the process of achieving it. Our method is subjected to oscillations due to the output of the LP solver used in (12). This is clear in Fig. 1 and 2 and has also impact on the system net benefit achieved towards the optimal (Fig. 3). Furthermore, it is obvious that the algorithm in [3] converges faster. This is not a general property but depends on the value of the parameter g employed for our method, which creates a trade-off between the convergence speed and the iterations needed for the oscillations
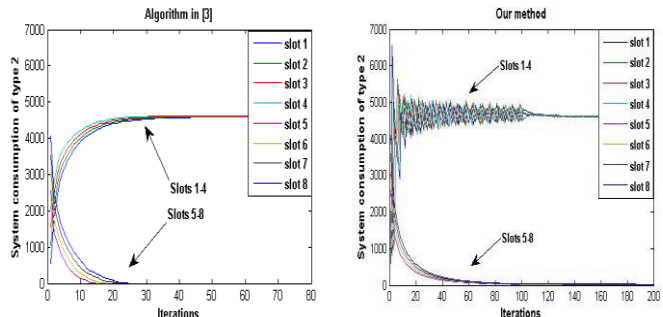

Figure 1. System consumption for type 2 appliances (left figure for the algorithm in [3] and right for the linearization method). In both cases, the load is gradually shifted at slots of no benefit for type 3 appliances.
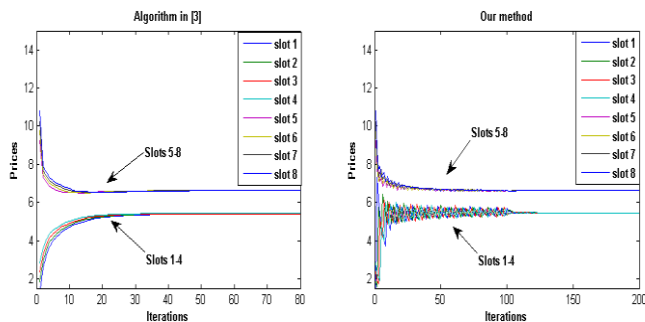

Figure 2. Prices per slot (left figure for the algorithm in [3] and right for the linearization method). Both approaches make the consumption smoother and reduce peak load demands.
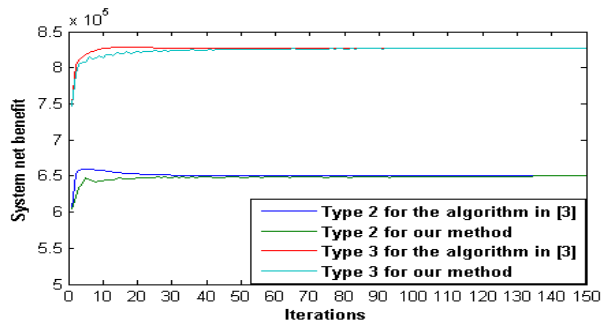

Figure 3. System net benefit for both approaches and types of appliances towards the optimal. The level achieved from our method is relatively close to that of [3] from the very first iterations.

"smooth-out" around the optimal point, as g^k tends to zero.

Fig. 2 shows the prices at each slot towards the convergence. Notice that optimal values are higher for slots 5-8 due to the weight of the utility function of type 3 appliances. Both approaches attain lower peak demands at different time periods, while balancing the need for power generation. The price vector at the convergence state has for both approaches almost equal values as follows:
$$p^*:= (5.43, 5.43, 5.43, 5.43, 6.62, 6.62, 6.62, 6.62)$$

In Fig. 3 we juxtapose the system net benefit achieved from the two methods at each iteration. The algorithm in [3] slightly outperforms ours and their difference decreases as our approach moves towards the optimal.
We additionally provide certain specific results concerning consumer 1, as described in Table I. The consumption of type 2 and type 3 appliances at the convergence state is almost equal for the two approaches, so we only present the results of our method.

TABLE I. Input values and results (of equation (12) at convergence state) for consumer 1. For both appliances the marginal utility equals the marginal cost.

Input A2	$w_{1,A2} = 10102 , \bar{Q}_{1,A2}^{min} = 1548 , \bar{Q}_{1,A2}^{max} = 2416$
Input A3	$w_{1,A3} := (0,0,0,0,3865,3818,3823,3845)$
Output A2	$\tilde{q}_{1,A2}^* := (469,464,462,467,0,0,0,0)$
Output A2	$\bar{Q}_{1,A2}^{min} < \bar{Q}_{1,A2}^* = 1862 < \bar{Q}_{1,A2}^{max}$
Output A3	$\tilde{q}_{1,A3}^* := (0,0,0,0,584,576,577,581)$

Notice that in both cases at convergence state $\gamma_{1,A2}^* \approx p^*(t)$, $t = 1,2,3,4$, while $\gamma_{1,A2}^* < p^*(t)$, $t = 5,6,7,8$, explaining the reason why the type 2 appliance of consumer 1 gradually refrains from consumption during these slots. This result applies for all consumers as it becomes obvious from Fig. 1. Additionally, $\gamma_{1,A3}^*(t) \approx p^*(t), t = 5,6,7,8$ meaning that for both appliances the marginal utility equals the marginal cost whenever they operate.

B. Evaluation of the Load Consolidation Method

In this section, we provide another numerical experiment, in which we apply both the linearization and the load consolidation methods. We evaluate the output compared to the same experiment, when only the linearization method is applied. We refer to the variables as "deterministic", in contrast to the case of applying also load consolidation, where we refer to these as "probabilistic".

We consider a variation of the system of Section V-A, with $N = 4000$ households, each operating one type 2 and one type 3 appliance, the utility functions of which are chosen as previously. The consumption of each type 2 appliance is initiated in a single slot (not the same for each consumer) while we keep the same ratio concerning $\hat{Q}_{A2}^0(t)$ among the slots $(\hat{Q}_{A2}^0(8) \approx \frac{8}{7} * \hat{Q}_{A2}^0(7))$, meaning that peak demanded slots arise also here. Additionally we assume that a portion of consumers (200 out of 4000) may operate the type 2 appliances only during the two latter slots of each day, i.e. $T_{i,A2} = \{7,8\}, i = 3801,3802,...,4000$. The consumption of each type 3 appliance is randomly and uniformly initiated in the range [600Wh, 620Wh] for the slots 5-8. Finally, we pick for both implementations the same pair of values for the parameter g as in Section V-A (0.25 and 0.03).

In Fig. 4 we juxtapose the deterministic and probabilistic prices for three slots. Notice that both implementations reduce the peak load demands and attain the same convergence speed, as it only depends on the value of g parameter (for the same input) and the load consolidation method has no impact on it. The oscillations of the deterministic system are abruptly smoothed out at iteration
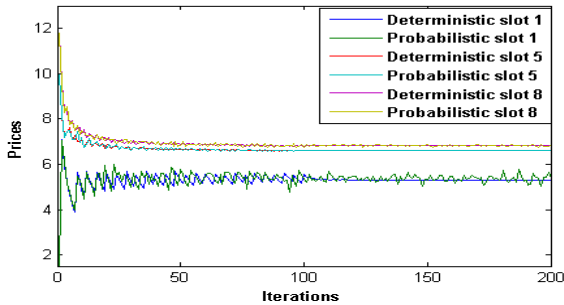

Figure 4. Deterministic and probabilistic prices for slots 1,5,8. The fluctuations at each slot depend on the load portion that type 2 appliances consume at it.
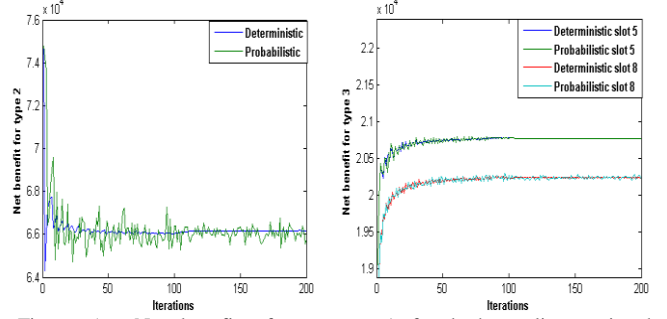

Figure 5. Net benefit of consumer 1 for both appliances in the deterministic and probabilistic case. The left figure applies to the type 2 appliance and the right to the type 3 one for slots 5 and 8.

TABLE II. Numerical results for consumer 1 under the deterministic and the randomized methods.

Net Benefit	Deterministic	Mean	%Dif.	St. D.
A2	66165	66054	-0.17	261.3
A3 at slot 5	20780	20780	0	~0
A3 at slot 8	20244	20245	~0	16.18

101, when we decrease the value of g, while the fluctuations of the probabilistic system remain due to the randomized reaction of type 2 appliances.

For the probabilistic system, notice that the fluctuations of $p(5)$ smooth out with iterations. This is expected, as type 2 appliances gradually shift their load to slots 1-4 (as in Section V-A) and tend to refrain from consumption at slot 5. The fluctuations of $p(1)$ are the most intense among the three slots, due to the fact that only type 2 appliances consume at this. In the case of slot 8, the type 3 appliances have a stabilizing role. Their consumption depends on the probabilistic prices (as a result of type 2 reactions), but as they alter their consumption pattern slowly, and consume the greatest portion of load at this slot, they keep the price value relatively close to the deterministic one.

Fig. 5 juxtaposes net benefit of consumer 1 at each iteration for both types of appliances. The above remarks concerning fluctuations at the various slots apply to this case as well. In Table II we provide numerical results for consumer 1 applying to Fig. 5, for iterations 120-200. The "Mean" and "Standard Deviation" (St. D.) columns refer to each probabilistic value. It is apparent that applying the consolidation method yields on the average almost the same user net benefit as the deterministic method for both types of appliances. Also, the level of the standard deviations is aligned with the intensity of fluctuations at each slot.

C. Adding Constraints across Appliances

In this section, we evaluate the linearization method (only) in the presence of constraints across the appliances, so we assume that the aggregate consumption drawn by each household at each slot is upper bounded by the maximum rate Q_i^{max}, i.e. $\sum_a q_{i,a}(t) \leq Q_i^{max} \ \forall t$, which is the same for all households: $Q_i^{max} = 350 \forall i$. We utilize the input parameters of Section V and adjust the initial consumption of Section V-A so as to satisfy the aforementioned constraint. This example is not comparable with the one of Section V-B, as in the presence of this constraint the load consolidation heuristic cannot be applied.

Fig. 6 shows the aggregate consumption of type 2 appliances (left) and the prices (right) for this system. Compared to the case of Section V-A, the additional constraint causes type 2 appliances to consume a portion of their load during slots 5-8. The prices are

equal for each slot $p^*(t) = 4.3\ \forall t$ because each user's aggregate per slot consumption is the same for all timeslots, and equal to the bound imposed by the additional constraint. These properties are further explained below on the output results for consumer 1.

In Table III we present the output results concerning the consumer 1, for comparison with those of Section V-A. In this case, type 2 appliance consumes also during slots 5-8 so as to satisfy the constraint concerning its minimum consumption value. At the convergence state $\gamma^*_{1,A3}(t) > \gamma^*_{1,A2}$, $t = 5,6,7,8$ meaning that the consumer gains more benefit from the type 3 consumption at these slots. This is the reason why type 2 appliance exactly consumes the requisite load in order to satisfy its minimum power requirements and the remaining is consumed by type 3 at each slot. Furthermore, notice that the load consumed from type 2 at each slot $t = 5,6,7,8$ is not arbitrary but depends on the corresponding weight of the utility function of type 3, so that the values of $\gamma^*_{1,A3}(t)$ are almost equal for all t. Finally, $\gamma^*_{1,A3}(t) > p^*(t)$ and $\gamma^*_{1,A2} > p^*(t)\forall t$, meaning that the consumer would gain extra benefit from further consumption at each slot from any appliance, but this is not possible due to the constraint concerning the total household consumption per slot. This is the reason why the aggregate per slot consumption equals Q_i^{max} at each slot.

I. CONCLUDING REMARKS AND FUTURE WORK

In this paper, we have built on the work in [3], where the authors propose a way to use dynamic pricing in the retail power market. Our contribution relies on proposing two heuristics giving a more practical version of the consumer's response to the dynamic prices. Simulations have shown that the proposed linearization method for the utility functions converges to the optimal state with "acceptable" net benefit loses at each intermediate iteration compared to the well defined gradient projection algorithm introduced in [3]. The speed of convergence and the oscillations "smooth-out" around the optimal state, both depending on the value of the relaxation parameter g, which has to be appropriately chosen according to the properties of each approach. Additionally, in the presence of appliances' constraints, our approach converges to the optimal state, while that of [3] does not appear to have been numerically evaluated for this case. Furthermore, the load consolidation method offers practical schedules for the consumers, while it causes negligible variations to the prices and the net benefit of each user compared to the case that it is not applied.

There are further important issues to be considered in order to align our methodology with real life needs and the market context. Regarding our approach, we should extend the load consolidation heuristic in the presence of constraints on the maximum consumption per slot. This issue does not seem to have a trivial solution and deserves further investigation. At the market level, we should clarify the conditions, over which an iterative computation is a practical DR approach, as the context defining the utility functions may differentiate faster than the iterations needed for the convergence. As a solution, we may assume that the procedure towards the optimal state is virtually applied before each day and the consumers actually consume only the optimal schedules discovered. In this case, consumers may respond by means of sophisticated software at the edges of the grid, which automatically adjusts their consumption schedule. Finally, in the case of being forced to deploy our method on top of flat rate pricing, we should identify the mechanism that "translates" the dynamic prices to personalized rebates for each consumer, while resulting at the same consumption outcome. We leave such issues for future work.
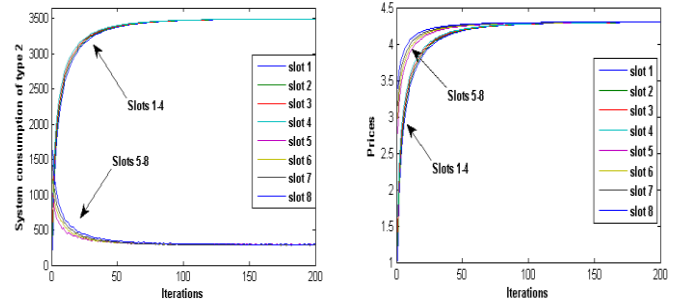

Figure 6. The left figure shows the system consumption for type 2 appliances, a portion of which is now during slots 5-8. The right figure shows the prices, the optimal values of which are determined by the additional constraint.

TABLE III. Input values and results (of equation (12) at convergence state) for consumer 1. The constraint across appliances determines the aggregate consumption for all slots. The consumption allocation among the appliances depends on the corresponding utility functions.

Input A2	$w_{1,A2} = 10102$, $\bar{Q}^{min}_{1,A2} = 1548$, $\bar{Q}^{max}_{1,A2} = 2416$
Input A3	$w_{1,A3} := (0,0,0,0,3865,3818,3823,3845)$
Output A2	$\tilde{q}^*_{1,A2} := (350,350,350,350,35,39,38,36)$
Output A2	$\tilde{Q}^*_{1,A2} = 1548 = \bar{Q}^{min}_{1,A2}$
Output A3	$\tilde{q}^*_{1,A3} := (0,0,0,0,315,311,312,314)$

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