An Optimization Framework for Effective Flexibility Management for Prosumers

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***Abstract*—Energy flexibility management can significantly support the smoother and more cost-effective green transformation of the energy mix. However, effective management of the flexibility of residential loads can only be achieved if users are successfully engaged into the process. In this paper, we propose an optimization framework that incorporates provision of different forms of monetary and non-monetary incentives to prosumers, i.e., rewards, lotteries, peer-pressure, for providing flexibility at specific time slots. Economic rewards are offered according to a simple, yet very powerful, linear incentives' function. Dynamic tariffs per time slot for purchasing and selling electricity are accommodated in this framework as well. The optimization problem of the DR aggregator is modeled as a cost-minimization one; its solution as a Stackelberg game is outlined for the case of full information on user-utility functions by the DR aggregator. Moreover, a distributed iterative algorithm is developed for solving the flexibility-management problem in the case where user-utility functions are not known to the aggregator. Numerical results show that this optimization framework is able to elicit the required flexibility from users at a minimum incentive cost, especially when monetary rewards are combined with peer pressure.**

***Index Terms*—rewards, peer-pressure, lottery, leader-follower game, dynamic electricity tariffs, feed-in tariffs**

I. Introduction

The liberalization of the electricity markets, energy crises creating economic pressures, and environmental regulations are all suggesting fewer traditional central power plants and the employment of more distributed and renewable energy resources (DER/RES) to address future energy needs. Towards this direction, policy makers and energy market participants concur that Demand Response (DR) for energy flexibility management and consumption curtailment is a critical resource for achieving an efficient and sustainable electricity system at a reasonable cost. This fact is reflected in the recent European regulation (Energy Efficiency Directive 2018/2002, Electricity Regulation 2019/943 and Directive on Electricity 2019/944). Flexibility management can enable the smooth introduction of DER/RESs to the electricity grid without impacting its stability by appropriately balancing supply with demand, and thus postponing huge investments in the grid infrastructure. Prosumers with rooftop photovoltaic panels (PVs) represent a rapidly developing and important category of potential flexibility providers. A long list of flexibility management solutions for residential consumers/prosumers have been proposed [1]. However, they mostly involve direct load control or fixed rule-based strategies for acquiring flexibility, and thus, user engagement in flexibility management programs cannot be taken for granted.

In this paper, we focus on implicit and explicit incentive schemes for flexibility management. To this end, we propose an innovative model for optimally aggregating a specific amount of flexibility from prosumers with PVs, involving dynamic pricing, feed-in tariffs and explicit incentives in the form of monetary rewards, peer pressure and lotteries. Flexibility provision can be either automated (upon user approval) or manual involving user action on shifting/curtailing electricity consumption. The overall problem of aggregating the desired flexibility at a specific time slot can be seen as a Stackelberg (i.e., leader-follower) game, between the DR aggregator and the users that play it by solving their respective problems. The aggregator's optimization problem is a cost minimization one for providing incentives to acquire the desired flexibility, while the problem of each prosumer is net benefit maximization involving flexibility incentives, comfort and electricity cost. For simple cases of user utility functions known to the aggregator (full-information case), an analytical solution to the overall flexibility management problem can be found. For the case that user utilities are not known to the aggregator, we develop a distributed iterative algorithm that finds optimal rewards for acquiring the desired flexibility from each prosumer based on gradient approximation. We extend our model to aggregate flexibility for multiple time slots and to include additional incentive schemes, namely peer pressure and lotteries. Based on numerical evaluation with real consumption data, we showcase that our model can effectively accommodate and combine different implicit or explicit incentive schemes. Also, we establish the effectiveness of the optimization framework to acquire the desired flexibility at the minimum cost for incentives, especially when peer pressure and monetary rewards are combined. Finally, we find that optimizing flexibility rewards jointly for multiple time slots is more cost-effective than performing myopic optimization of rewards for each

This work has been funded by the EU project iFLEX (grant no. 957670).

978-1-6654-3254-2/22/$31.00

different time slot consecutively.

II. Related Work

There is significant related work on energy flexibility and Demand Response (DR). A recent review [1] has presented a long list of flexibility quantification metrics, and control and optimization strategies for flexibility management for residential users. 51% of strategies in [1] employ direct load control out of which 72% use optimal controls, while 28% use rule-based controls, e.g., for water heaters, shift the water heating power demand from peak periods to off-peak periods. Our approach focuses on implicit and explicit incentives for prosumers, so that the provision of flexibility (either based on direct load control or not) is beneficial for prosumers.

An optimization problem to satisfy distribution system operator (DSO) balancing requests on local flexibility markets has been proposed in [2]. It has been formulated as a mixed integer linear problem (MILP) one for cost minimization taking into account flexibility of various residential loads, costs for providing various types of flexibility, e.g., costs for reducing output of distributed energy resources, costs for battery charging/discharging, and prices (i.e., rewards) for load shifting/curtailment to residential customers. Real-time electricity tariffs and different incentive schemes are not considered, while direct load control of all flexible loads is assumed in [2], as opposed to the present work. A flexibility-constrained energy management model has been proposed in [3] for smart homes with PVs and batteries. The flexibility constraint limits the ramp rate (i.e., difference in purchased/sold power from the smart home to the main grid between two consecutive time slots) to increase the flexibility of the power system. The home energy management problem, modelled as MILP, takes into account prices for purchasing/selling electricity and efficiency parameters for the PV and the energy storage. However, the approach in [3] does not take into account user discomfort from the modified energy-consumption schedule or any incentive schemes other than the feed-in tariffs, as opposed to the present work. A model (MILP) for the energy management of a community of houses has been proposed in [4], which involves trading of energy among the houses so that the cost of purchasing power from the main grid especially in peak hours is minimized. An energy flexibility management framework, based on the concept of multi-energy lattice has been proposed in [5]. Energy flexibility arises both from single- and cross-layer energy balancing. The problem is formulated as a two-step MILP problem, based on multi-energy baselines to cope with the energy demand across multiple energy vectors, and flexibility margins and economic convenience to offer different frequency control ancillary services.

Also, in the DR literature, a real-time pricing (RTP) DR scheme has been proposed in [6], where, given optimal prices, consumers take consumption-scheduling decisions aiming at individual net benefit maximization. The utility company has to choose the optimal real-time prices to set, so that the social welfare of the consumers is maximized. The problem is solved based on a distributed iterative algorithm between the utility company and the consumers. Social welfare maximization of consumers is also the goal of the operator in [7], where the optimal strategy is to set prices equal to the marginal cost of supply. In [7], they derive a convergent distributed algorithm based on dual decomposition method. A cost-minimizing operator that provides incentives to residential users to shift their demands through dynamic pricing has been considered by [8]. In [9], they aim to design an optimum DR scheme that not only reduces costs and improves reliability, but also increases customer acceptance of the DR program by limiting price volatility. Both price-based and incentive-based DR programs are considered in [9]; especially for the latter, they calculate the required energy load change, the corresponding adequate incentive value and the best timing to implement DR. In [10], they propose to compensate consumers for their discomfort due to modifications in their consumption patterns in inclusive DR (iDR) schemes. It is argued in [10] that whenever participating in DR a user should be offered incentives at least as high as his reduction in net benefit, that is loss of utility (due to discomfort) minus savings in the energy bill. Specific behavioral characteristics delineating users' behavior (e.g. altruism) are incorporated in users' utility functions in [11] employ a customer targeting approach, so that the DR designer constructs ADR contracts with appropriate personalized rewards for customers to (a) enroll in them in the first place and (b) extend/renew their ADR contracts based on customer feedback on their satisfaction from ADR contracts. Overall, the DR designer in [11] aims to minimize the total endowment for achieving the needed energy curtailment, while maintaining the customer satisfaction ratio over a certain threshold.

III. The Context

We consider the case of an aggregator or a utility acting as a Balancing Responsible Party (BRP), referred to as DR aggregator. The DR aggregator is responsible to provide adequate DR signals to consumers/prosumers, so as to aggregate a specific amount of flexibility in one or several slots from a community of prosumers in a geographical area. We assume for the time being that the requested flexibility is positive, i.e., either electric load reduction or electric power injected into the grid. The requested amount of flexibility is assumed to be predicted based on DER/RES power generation or electricity demand forecasts. Home Energy Management Systems (HEMSs) are assumed to be installed at all prosumer/consumer premises. In particular, HEMS provide power-consumption readings from individual devices at residential premises, e.g., heat pumps and boilers, and readings from sensors for comfort (i.e., temperature, humidity). When available, PVs generate electricity that can be either injected directly into the grid or consumed locally upon production, since we assume that there is no electricity storage capability. The customer pays only for the total net power consumption in each time slot of the billing period (please see below), or he is compensated according to feed-in tariffs in case of excess local production per slot. No other kind of selling of electricity actually occurs.

Flexibility is expected to be provided by consumers/prosumers based on DR signals that include high network/retail tariffs for specific hours announced one day ahead. The users are expected to modify their electricity-consumption schedules in response to these higher tariffs (implicit DR), so as to minimize their electricity bill, and thus offer flexibility. In addition, DR incentive signals may include explicit incentives, such as monetary rewards, tokens for participating in lotteries, or the relative performance of the user in terms of flexibility as a form of peer pressure.

The users practically cannot opt-out. However, they are free to choose their own self-optimizing way to react to DR signals, by adjusting (or not) their electricity consumption schedule accordingly. Hence, while they cannot declare opting-out, they may opt out in practice by not responding to DR signals. Users are able to choose which activities involving electricity consumption to shift in time or cancel being also aided by the HEMSs. The flexibility offer can be estimated based on current load profiles of the users. This offer is declared ahead of the DR event, and can be verified by means of the HEMSs.

IV. The Model

We consider a set $\mathcal{N}$ of N consumers/prosumers. Each day is divided into T time slots, indexed by t. For each consumer $n \in \mathcal{N}$, we denote as $\mathbf{x}_{n,0}$ the daily vector of baseline energy consumption per time slot prior to DR. Moreover, we assume that some customers possess solar panels. For each such prosumer n, we denote as $\mathbf{w}_n = \{w_n^t\},\ \forall t \in T$ the energy generation vector of his solar panels.

A. DR Aggregator's problem

We denote as x_n^τ the flexibility offered by consumer n at time slot τ. We assume that each consumer n is compensated according to a linear incentives' policy. That is, he receives by the DR aggregator incentives r_n per flexibility unit provided thereto. Then, the objective of the DR aggregator is to select the appropriate set of consumers and the flexibility x_n^τ to be asked by each of them so that the required flexibility X^τ is met at a specific time slot of interest τ with the minimum total amount of incentives offered. $\mathbf{x}_{n,1} = \{x_{n,1}^t\},\ \forall t \in \mathcal{T}$ is the updated daily consumption schedule for consumer n subject to changes $\mathbf{x}_n = \mathbf{x}_{n,0} - \mathbf{x}_{n,1}$ throughout the day due to the flexibility event at time slot τ. Then, the DR aggregator's optimization problem can be formulated as follows:

$$\textbf{Minimize:} \sum_{n \in \mathcal{N}} r_n y_n x_n^\tau \tag{1}$$

$$\textbf{s.t.} \sum_{n \in \mathcal{N}} y_n x_n^\tau \geq X^\tau \tag{2}$$

$$\begin{aligned} y_n \Big(& r_n x_n^\tau + U_n(\mathbf{x}_{n,0} - \mathbf{x}_n) - U_n(\mathbf{x}_{n,0}) \\ & - \sum_{t \in \mathcal{T}, x_{n,0}^t - x_n^t \geq w_n^t} (x_{n,0}^t - x_n^t - w_n^t)\psi^t + \sum_{t \in \mathcal{T}, x_{n,0}^t \geq w_n^t} x_{n,0}^t \psi^t \\ & + \sum_{t \in \mathcal{T}, 0 < x_{n,0}^t - x_n^t < w_n^t} (w_n^t - x_{n,0}^t + x_n^t)\phi^t \\ & - \sum_{t \in \mathcal{T}, 0 < x_{n,0}^t < w_n^t} (w_n^t - x_{n,0}^t)\phi^t \Big) \geq 0,\ \forall n \in \mathcal{N} \end{aligned} \tag{3}$$

$$x_n^\tau \leq x_{n,0}^\tau + w_n^\tau,\ \forall n \in \mathcal{N} \tag{4}$$

where $y_n \in \{0, 1\}$ is the decision variable for targeting user n. ψ^t is the electricity unit price at time slot t, while ϕ^t is the feed-in tariff for injecting renewable energy into the network, which can be different per time slot t. Recall that no other kind of energy trading is assumed to take place. Constraint (3) is an incentive compatibility condition, which ensures that each targeted user will indeed offer the flexibility requested thereby, because he is better-off in terms of net benefit than by not participating in DR. Also, constraint (4) is a feasibility condition for flexibility provision that dictates flexibility per user to be upper bounded by the sum of his baseline energy consumption and his solar energy production at time slot τ.

To simplify our problem, we henceforth consider a simpler DR incentives policy. In particular, we assume that all users are offered the same incentive r per flexibility unit, while all users may offer flexibility (i.e., there is no targeting). This amounts to a more inclusive DR incentives policy, where all consumers individually decide how to participate or not, subject to the common per unit incentives being publicly announced by the DR aggregator. In this case, the formulation of the DR aggregator's optimization problem becomes:

$$\textbf{Minimize:} \sum_{n \in \mathcal{N}} r x_n^\tau \tag{5}$$

$$\textbf{s.t.}\ (4),\ \sum_{n \in \mathcal{N}} x_n^\tau \geq X^\tau \tag{6}$$

$$\begin{aligned} & r x_n^\tau + U_n(\mathbf{x}_{n,0} - \mathbf{x}_n) - U_n(\mathbf{x}_{n,0}) \\ & - \sum_{t \in \mathcal{T}, x_{n,0}^t - x_n^t \geq w_n^t} (x_{n,0}^t - x_n^t - w_n^t)\psi^t + \sum_{t \in \mathcal{T}, x_{n,0}^t \geq w_n^t} x_{n,0}^t \psi^t \\ & + \sum_{t \in \mathcal{T}, 0 < x_{n,0}^t - x_n^t < w_n^t} (w_n^t - x_{n,0}^t + x_n^t)\phi^t \\ & - \sum_{t \in \mathcal{T}, 0 < x_{n,0}^t < w_n^t} (w_n^t - x_{n,0}^t)\phi^t \geq 0,\ \forall n \in \mathcal{N} \end{aligned} \tag{7}$$

This optimization problem can be solved by the aggregator w.r.t. r, $\mathbf{x_n}, \forall \mathbf{n} \in \mathcal{N}$ when the DR aggregator has full information concerning the utility functions, i.e., when $U_n(\cdot)$ is known to the aggregator for each customer n, as explained in Section IV-C. Otherwise, we have to resort to a distributed and iterative solution, which is described in Section IV-D. In that case, the aggregator solves the optimization problem of equation (5) considering only the constraint (6), while the user solves his own optimization problem, which is introduced in Section IV-B.

B. User's problem

We assume that users are offered by the DR aggregator incentives r per flexibility unit at time slot τ. Then each user n has to select his optimal flexibility vector $\mathbf{x}_n$, by solving the following problem:

$$\begin{aligned}&\textbf{Maximize: } rx_n^\tau + U(\mathbf{x}_{n,0} - \mathbf{x}_n) - U_n(\mathbf{x}_{n,0})\\ &- \sum_{t\in\mathcal{T}, x_{n,0}^t - x_n^t \geq w_n^t} (x_{n,0}^t - x_n^t - w_n^t)\psi^t + \sum_{t\in\mathcal{T}, x_{n,0}^t \geq w_n^t} x_{n,0}^t\psi^t\\ &+ \sum_{t\in\mathcal{T}, 0< x_{n,0}^t - x_n^t < w_n^t} (w_n^t - x_{n,0}^t + x_n^t)\phi^t\\ &\qquad - \sum_{t\in\mathcal{T}, 0< x_{n,0}^t < w_n^t} (w_n^t - x_{n,0}^t)\phi^t \end{aligned} \tag{8}$$

For user n to participate in DR, two conditions should hold:

- Individual Rationality (IR): The net benefit from participating in DR should be positive.
- Incentive Compatibility (IC): The net benefit from participating in DR should be higher than that when not participating, or equivalently the difference of these two net benefit values should be non-negative; this amounts to condition (7).

Since $\mathbf{x}_{n,0} > 0$, for each user n, his net benefit when not participating in DR is positive. Thus, IR and IC conditions are concurrently met when the IC condition (7) is true.

Our analysis to follow in Section IV-C is applicable to any increasing and differentiable user utility function. Nevertheless, for simplicity, we employ the following utility function $U(\cdot)$:

$$U_n(\mathbf{x}_n) = \sum_{t\in\mathcal{T}} \beta_n^t (x_{n,0}^t - x_n^t)\,, \tag{9}$$

where $\beta_n^t = x_{n,0}^t / \max\{\mathbf{x}_n\}$. According to this function, a reduction (resp. increase) in energy consumption at a certain time slot results in loss (resp. gain) of comfort, and thus of utility, for the user. These utility deviations are summed in a weighted fashion over the entire time period $\mathcal{T}$ considered, with a normalized weight per slot that is proportional to the baseline electricity consumption during that time slot, expressing the relative importance that the user assigns to consuming energy at that particular time of the day.

C. Equilibrium

The aforementioned problems, in fact, can jointly be seen as a Stackelberg (i.e. leader-follower) game: the DR aggregator moves first to select incentives r and each user n follows by selecting his flexibility $\mathbf{x}_n$. The overall optimization problem (comprising the DR aggregator's problem and those of the users) can be solved by means of backwards induction. That is, the user's problem is solved first for each user n, to find the optimal flexibility $\mathbf{x}_n$, assuming that optimally chosen incentives r^*, applicable for the entire population of users, have been announced by the DR aggregator. Then, the DR aggregator's problem could be solved, if the dependence of $\mathbf{x}_n$, $\forall n \in \mathcal{N}$ to r^*, were known expressed in closed form. However, this is a very restrictive assumption. Hence, we resort to a distributed iterative algorithm for deriving the desired equilibrium point.

D. Distributed Algorithm

As already explained, in the case where user utility functions are not known, then the DR aggregator and the user should solve their individual problems, i.e., (5) s.t. (6) for the DR aggregator and (8) for the user respectively. For this purpose, we introduce the following distributed iterative approach: The DR aggregator and the consumers jointly compute an equilibrium based on a gradient approximation algorithm [12], where (i) the DR aggregator sets the reward per flexibility unit and (ii) each prosumer solves his own maximization problem in response.

At the beginning of each round k, the DR aggregator announces the per unit of flexibility incentives r. Each user n updates his offered flexibility $\mathbf{x}'_n$ according to the formula below and announces it to the DR aggregator.

$$\begin{aligned} x_n^{t,k+1} = \min\{&x_{n,0}^t,\, x_n^{t,k} + \xi(r^k + (-\beta^t) +\\ &\psi^t \mathbf{1}(x_{n,0}^t - x_n^t - w^t) - \phi^t \mathbf{1}(w^t - x_{n,0}^t + x_n^t)\} \end{aligned} \tag{10}$$

$\mathbf{1}(\cdot)$ is an indicator function, which equals 1 if its argument is greater than zero, or 0 otherwise. Then, DR aggregator updates the per unit of flexibility incentives r according to the formula below.

$$r^{k+1} = \max\left\{ r^k + \xi\left(X^\tau - \sum_{n\in\mathcal{N}} x_n^\tau\right),\, 0\right\} \tag{11}$$

At the end of each round, the DR aggregator sets $r = r'$ and each customer $n \in \mathcal{N}$ sets $\mathbf{x}_n = \mathbf{x}'_n$. This iterative process stops when the values of r and $\mathbf{x}_n, \forall n \in \mathcal{N}$ converge, provided that the desired flexibility at the time τ has been aggregated.

E. Flexibility in Multiple Slots

We now consider the case that the aggregator requests flexibility in multiple time slots $\mathcal{S}$. One may argue that this problem can be solved by consecutively solving problem (5) for each of the time slots of interest. However, such a solution would only be suboptimal, as the flexibility provided separately per time slot would only be "myopic" without being able to find the updated consumption schedules that joint satisfy the flexibility objectives. The multi-slot aggregator problem is given by:

$$\textbf{Minimize: } \sum_{\tau\in\mathcal{S}}\sum_{n\in\mathcal{N}} rx_n^\tau \tag{12}$$

$$\textbf{s.t. } \sum_{n\in\mathcal{N}} x_n^\tau \geq X^\tau\,, \forall\tau\in\mathcal{S} \tag{13}$$

$$r\sum_{\tau\in\mathcal{S}}(x_n^\tau)+U_n(\mathbf{x}_{n,0}-\mathbf{x}_n)-U_n(\mathbf{x}_{n,0})$$
$$-\sum_{t\in\mathcal{T},x_{n,0}^t-x_n^t\geq w_n^t}(x_{n,0}^t-x_n^t-w_n^t)\psi^t+\sum_{t\in\mathcal{T},x_{n,0}^t\geq w_n^t}x_{n,0}^t\psi^t$$
$$+\sum_{t\in\mathcal{T},0<x_{n,0}^t-x_n^t<w_n^t}(w_n^t-x_{n,0}^t+x_n^t)\phi^t$$
$$-\sum_{t\in\mathcal{T},0<x_{n,0}^t<w_n^t}(w_n^t-x_{n,0}^t)\phi^t\geq 0,\ \forall n\in\mathcal{N} \quad (14)$$

Again, in case of full information on user utilities by the DR aggregator, this problem can be seen as a Stackelberg game between the DR aggregator and the users and it can be solved similarly to the approach described in Section IV-C. In case of hidden information on user utilities, a distributed iterative approach similar to that of Section IV-D can be followed to find the solution.

V. Additional Incentives

A. Peer Pressure

One behavioral driver for reducing power consumption is peer pressure. Peer pressure can be exercised for flexibility management simply by privately announcing to the user the relative performance percentile to which he falls according to his offered flexibility in a number of previous DR events. We can assume that the user enjoys some personal *satisfaction* h_i from being ranked first. The higher the flexibility offered by the user at the time duration of interest, the higher the probability that this user is ranked first. We approximate that probability below.

First, we remind some material from the theory of order statistics [13]. Let $X_1,\ldots,X_{N-1}$ be $N-1$ independent and identically distributed (i.i.d.) random variables. In our case, the random variable X_j denotes the energy flexibility of user j. The order statistics $X_{(1)},X_{(2)},\ldots,X_{(N-1)}$ are also random variables, defined by sorting the realizations of $X_1,\ldots,X_{N-1}$ in non-decreasing order. Namely, for each realization ω, we arrange the sample values $X_1(\omega),\ldots,X_{N-1}(\omega)$ in non-decreasing order, $X_{(1)}(\omega)\leq X_{(2)}(\omega)\leq\ldots\leq X_{(N-1)}(\omega)$, where $(1),(2),\ldots,(N)$ denote that permutation of indices $1,2,\ldots,N-1$ for which the random variables X are ordered. Thus, we have:

$$\begin{aligned} X_{(1)}&=\min\{X_1,\ldots,X_{N-1}\}\\ &\vdots\\ X_{(N-1)}&=\max\{X_1,\ldots,X_{N-1}\}\,. \end{aligned} \quad (15)$$

For a user n to be ranked first, it is necessary and sufficient that his flexibility x if higher than the maximum flexibility of all $N-1$ other users. For a fixed value of x, this probability is given by:

$$\begin{aligned} F_{X_{(N-1)}}(x)&=Prob(\max\{X_1,\ldots,X_{N-1}\}\leq x)\\ &=|F_X(x)|^{N-1}\,, \end{aligned} \quad (16)$$

where $F_X(\cdot)$ is the common cumulative distribution function (CDF) of the variables $X_1,\ldots,X_{N-1}$. This CDF can be estimated based on prior flexibility performance of the users. If the baseline consumption of the users are identically distributed random variables, then a simple approximation, this CDF can be assumed equal to that of the baseline consumption of the users in the flexibility slot scaled by $X/\sum_{n\in\mathcal{N}}x_{n,0}$, which is the factor by which the total consumption is discounted due to offering of flexibility. The term $F_{X_{(N-1)}}(\widetilde{x}_n)\cdot h_n$ is the average benefit of a user n from being ranked first when exerting flexibility $\widetilde{x}_n$. This term should be added to the user problem (8) to express the incentives by the mechanism of peer pressure.

B. Lotteries

According to prospect theory [14], when presented with alternatives that involve risk and uncertainty, people tend to prefer higher gains with small probability than equivalent (on the average) gains with certainty. Flexibility rewards cannot always be easily implemented due to significant changes required to the billing system and complexity for their inclusion in the accounting records of the utility company or aggregator. Alternative to the flexibility rewards, albeit with simpler implementation in practice, could be the organization of weekly/monthly lotteries for flexibility with a certain price of higher value V. The higher the participation of the user to flexibility events and the higher the flexibility offered, the higher the probability to win the lottery. This could be achieved in practice by providing a number of lottery tickets to each user that is proportional to the flexibility he offers over a certain period. Then, a ticket would be drawn from the lottery to determine the winner of the particular period. A new lottery would subsequently be started for the next period. The probability for a user to win the lottery is proportional to his individual flexibility over the sum of total flexibility provided by all users. However, according to prospect theory, since the probability p to win a high-valued prize is low, the satisfaction of a user from this prize is given by $\pi(p)\cdot V>p\cdot V$; this is referred to as "hope of large gain". As compared to the reward per flexibility unit, a lottery can be more motivating for flexibility management for the same incentive cost.

VI. Evaluation

We employed MATLAB and the REDD dataset [15] to evaluate the proposed optimization framework. The baseline consumption of the houses is depicted in Fig. 1a. We assume that the requested flexibility is 2KWh at the time slot of 12pm-1pm. We consider that the off-peak tariff per electricity unit is 0.1€/KWh, while the peak tariff, employed between 12pm-1pm, becomes 0.4€/KWh. We assume that the utility function (9) expresses the satisfaction for user n from his consumption schedule. We assume the parameter $\xi=0.01$ in the distributed algorithm of Section IV-D.

For peak-tariff pricing at the time slot of the desired flexibility without any flexibility rewards ($r=0$), the optimal solution to problem (5) is depicted in Fig. 2a, while its decentralized solution is depicted in Fig. 2b. Observe that flexibility distributions are aligned, i.e., Pearson correlation was found to be 0.75. However, in the decentralized solution, there are some rebound effects

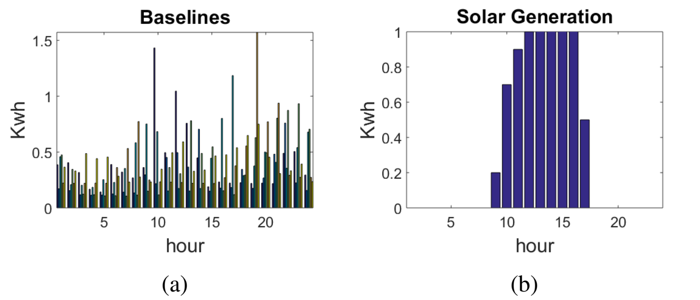


Fig. 1: (a) The baseline consumption. (b) PV power generation hourly schedule per prosumer.

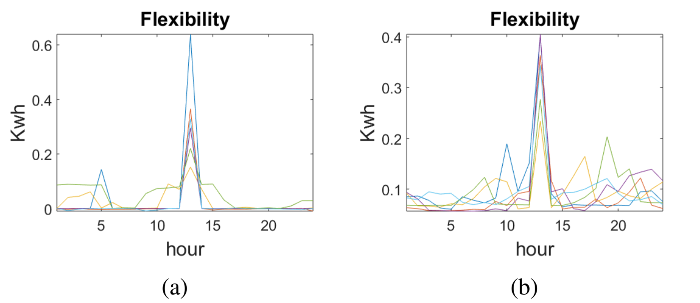


Fig. 2: Peak-tariff pricing: (a) Optimal flexibility per user. (b) Flexibility per user with the distributed algorithm.

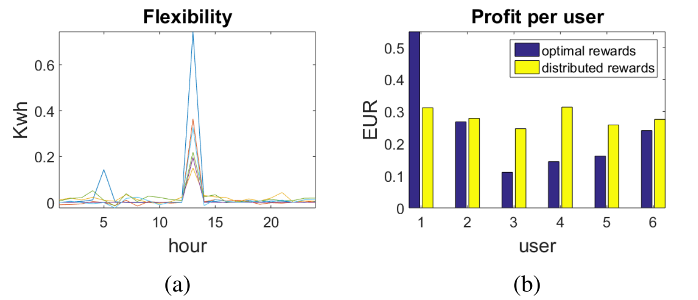


Fig. 3: Flexibility rewards: (a) Optimal flexibility per user; (b) Flexibility profit per user with optimal and suboptimal (distributed) rewards.

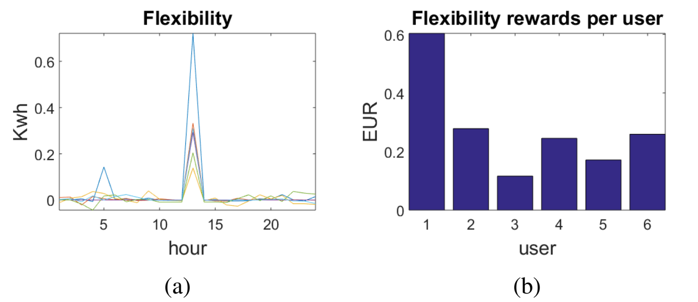


Fig. 4: PV power generation: (a) Optimal flexibility per user, (b) given specific flexibility rewards.

regarding consumption in the other time slots. The decentralized solution has converged in 46 iterations.

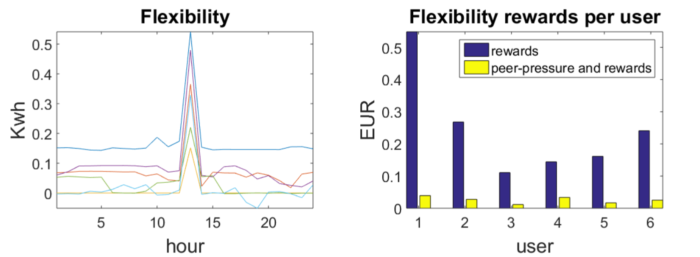
If no peak-tariff pricing is employed, but flexibility rewards are in order, then the desired flexibility is obtained (cf. Fig. 3a) at a cost for incentive rewards (cf. Fig. 3b). The total amount of rewards given is 1.47€ for this flexibility event. The flexibility distribution from optimal rewards is almost the same with that of Fig. 2a, i.e., Pearson correlation coefficient is 0.974, while total flexibility is 2KWh in both cases, which means that the users offer the same flexibility regardless of the exact incentive mechanism employed, i.e., peak-tariff pricing or rewards. In Fig. 3b, we also compare the optimal rewards with those resulting from the distributed algorithm (that converged in 36 rounds). The distribution of rewards per user from the distributed algorithm is smoother than the optimal one, while the total cost of incentives is slightly higher than optimal, i.e., 1.68€, as expected.

We now consider the case of solar power generation according to Fig. 1b. We assume that net metering is employed, i.e., the feed-in tariff is 0.1 €/KWh. When flexibility rewards are employed instead of peak-tariff pricing, then the flexibility provided by the users and their respective rewards are depicted in Fig. 4a and Fig. 4b respectively. The total required flexibility rewards here are higher, i.e. 1.67€; otherwise, the users would prefer the feed-in tariff for their PV energy instead of providing it to the DR aggregator.

We also evaluate the alternative incentive mechanisms in terms of effectiveness for flexibility management. Employing only peer pressure and assuming satisfaction from being ranked first $h_n = 1, \forall n \in \mathcal{N}$, the flexibility offered by the users in the time slot of interest is depicted in Fig. 5a. Combining peer-pressure with rewards results in lower rewards for aggregating the same amount of flexibility, as illustrated in Fig. 5b.

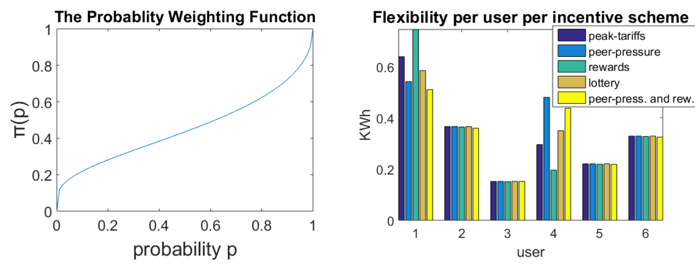
Moreover, we employed lotteries alone assuming the lottery prize equal to the total cost for rewards per flexibility event, i.e., 1.47€, so that a fair comparison can be made. The probability weighting function (i.e., $e^{-(-ln(x))^{0.5}}$) [16] that we employed to express the user prospect with respect to the probability to win is depicted in Fig. 6a. In Fig. 6b, we illustrate the flexibility offered by each user with the different forms of incentive schemes (at the same cost for rewards and lotteries).

Next, we consider the case where the aggregator aims to find the optimal rewards in order to gather flexibility 2KWh at each of three different time slots, namely 11am-12pm, 12-1pm and 1-2pm hours. Employing the aggregate optimization approach (12), we find that the optimal reward per flexibility unit is 0.57 €/KWh. As depicted in Fig. 7, the optimal rewards per user for the three time slots with the multi-slot optimization framework are lower than those that are found to be needed by employing the flexibility optimization approach (5) for each of the three time slots consecutively. Separate consecutive flexibility optimizations per time slot give optimal rewards per flexibility unit



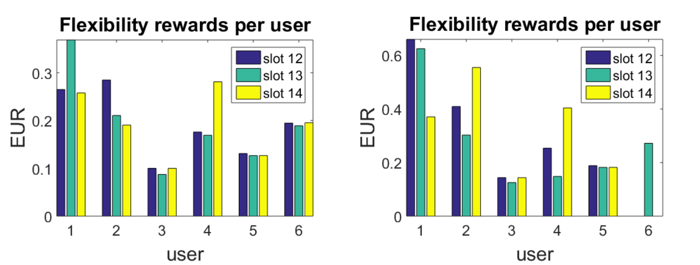
(a) (b)

Fig. 5: (a) Optimal flexibility per user with peer pressure. (b) Flexibility rewards per user when rewards are combined with peer pressure.



(a) (b)

Fig. 6: (a) A probability weighting function π. (b) Comparison of optimal flexibility per user with different incentive schemes.



(a) (b)

Fig. 7: (a) Optimal flexibility rewards employing (a) multi-slot optimization, (b) consecutive myopic optimizations per time slot.

of 0.63€/KWh, 0.73€/KWh and 0.82€/KWh respectively for the three time slots. Therefore, as expected, myopic optimization of the incentives for flexibility per time slot is outperformed by the multi-slot optimization approach.

VII. Conclusion

In this paper, we proposed an innovative optimization framework for flexibility management of prosumers with rooftop PVs. Our model can accommodate implicit and explicit incentive schemes, namely dynamic pricing, monetary rewards, lotteries and peer pressure for successfully engaging prosumers into flexibility management, as well as multiple non-necessarily consecutive time slots for flexibility requests. The overall optimization problem can be seen as a Stackelberg game that can be analytically solved by the flexibility aggregator for simple cases of user utility functions that are known to it. We also proposed a distributed iterative algorithm between the flexibility aggregator and the prosumers to solve the problem in case of hidden information on user utility functions. By means of numerical evaluation with real data, we demonstrated the effectiveness of the model to find optimal user consumption schedules that provide to the aggregator the desired flexibility in specific time slots at the minimum cost for incentives, when monetary incentives are employed alone or combined with non-monetary ones. As a future work, we plan to accommodate in the model electricity storage capability at the prosumer premises.

References

[1] H. Li, Z. Wang, T. Hong, and M. A. Piette, "Energy flexibility of residential buildings: A systematic review of characterization and quantification methods and applications," *Advances in Applied Energy*, vol. 3, p. 100054, 2021.

[2] P. Olivella-Rosell, E. Bullich-Massagué, M. Aragüés-Peñalba, A. Sumper, S. Ødegaard Ottesen, J.-A. Vidal-Clos, and R. Villafáfila-Robles, "Optimization problem for meeting distribution system operator requests in local flexibility markets with distributed energy resources," *Applied Energy*, vol. 210, pp. 881–895, 2018.

[3] S. Bahramara, "Robust optimization of the flexibility-constrained energy management problem for a smart home with rooftop photovoltaic and an energy storage," *Journal of Energy Storage*, vol. 36, p. 102358, 2021.

[4] S. M. Hakimi and A. Hasankhani, "Intelligent energy management in off-grid smart buildings with energy interaction," *Journal of Cleaner Production*, vol. 244, p. 118906, 2020.

[5] E. Corsetti, S. Riaz, M. Riello, and P. Mancarella, "Modelling and deploying multi-energy flexibility: The energy lattice framework," *Advances in Applied Energy*, vol. 2, p. 100030, 2021.

[6] N. Li, L. Chen, and S. H. Low, "Optimal demand response based on utility maximization in power networks," in *IEEE Power and Energy Society General Meeting*, 2011.

[7] L. Chen, N. Li, L. Jiang, and S. H. Low, *Optimal Demand Response: Problem Formulation and Deterministic Case*. New York, NY: Springer New York, 2012, pp. 63–85.

[8] L. Gkatzikis, T. Salonidis, N. Hegde, and L. Massoulié, "Electricity markets meet the home through demand response," in *IEEE Conference on Decision and Control (CDC)*, 2012.

[9] A. Asadinejad and K. Tomsovic, "Optimal use of incentive and price based demand response to reduce costs and price volatility," *Electric Power Systems Research*, vol. 144, pp. 215–223, 2017.

[10] V. Chandan, T. Ganu, T. K. Wijaya, M. Minou, G. Stamoulis, G. Thanos, and D. P. Seetharam, "Idr: Consumer and grid friendly demand response system," in *Proceedings of the 5th International Conference on Future Energy Systems*, ser. e-Energy '14. New York, NY, USA: Association for Computing Machinery, 2014.

[11] T. G. Papaioannou, G. D. Stamoulis, and M. Minou, "Personalized feedback-based customer incentives in automated demand response," in *IEEE International Conference on Communications, Control, and Computing Technologies for Smart Grids (SmartGridComm)*, 2018.

[12] L. Hannah, "Stochastic optimization," *International Encyclopedia of the Social & Behavioral Sciences*, vol. 2, 2015.

[13] H. A. David, "Order statistics," in *International Encyclopedia of Statistical Science*, M. Lovric, Ed. Springer Berlin Heidelberg, 2011, pp. 1039–1040.

[14] D. Kahneman and A. Tversky, "Prospect Theory: An Analysis of Decision under Risk," *Econometrica*, vol. 47, no. 2, pp. 263–291, 1979.

[15] J. Kolter and M. Johnson, "REDD: A Public Data Set for Energy Disaggregation Research," *Artif. Intell.*, vol. 25, 2011.

[16] D. Prelec, "The probability weighting function," *Econometrica*, vol. 66, no. 3, pp. 497–528, 1998.